Math 314/814, Section 4

Quiz number 6 Solution

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

Use the row reduction to find the determinant of the matrix

$$A = \begin{pmatrix} -1 & 1 & 2 & 1 \\ 1 & 2 & -2 & 3 \\ 3 & 2 & -3 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}.$$

As usual, there are lots of paths through the row reduction; here is one:

$$A = \begin{pmatrix} -1 & 1 & 2 & 1 \\ 1 & 2 & -2 & 3 \\ 3 & 2 & -3 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \ (1) \rightarrow \begin{pmatrix} -1 & 1 & 2 & 1 \\ 0 & 3 & 0 & 4 \\ 0 & 5 & 3 & 4 \\ 0 & 2 & 3 & 2 \end{pmatrix} \ (2) \rightarrow \begin{pmatrix} -1 & 1 & 2 & 1 \\ 0 & 2 & 3 & 2 \\ 0 & 5 & 3 & 4 \\ 0 & 3 & 0 & 4 \end{pmatrix}$$

$$(3) \rightarrow \begin{pmatrix} -1 & 1 & 2 & 1 \\ 0 & 1 & 3/2 & 1 \\ 0 & 5 & 3 & 4 \\ 0 & 3 & 0 & 4 \end{pmatrix} \ (4) \rightarrow \begin{pmatrix} -1 & 1 & 2 & 1 \\ 0 & 1 & 3/2 & 1 \\ 0 & 0 & -9/2 & -1 \\ 0 & 0 & -9/2 & 1 \end{pmatrix}$$

$$(5) \rightarrow \begin{pmatrix} -1 & 1 & 2 & 1 \\ 0 & 1 & 3/2 & 1 \\ 0 & 0 & -9/2 & -1 \\ 0 & 0 & 0 & 2 \end{pmatrix} = R$$

For steps (1), (4), and (5) we add multiples of one row to another, which does not change the determinant [the corresponding elementary matrix we multiply by has determinant 1]. For step (2) we swap two rows, which multiplies the determinant by -1. And for step (3) we multiply a row by $1/2$, which multiplies the determinant by $1/2$. So we find that

$$\det(R) = (-1)(1)(-\frac{9}{2})(2) = 9 = \det(E_{2,4})\det(E_2(1/2))\det(A)$$
$$= (-1)(\frac{1}{2})\det(A) = -\frac{1}{2}\det(A), \text{ so}$$

$$\det(A) = -2\det(R) = (-2)(9) = -18\ .$$

Here is one with no row swaps or rescaling (so the det is the productof the diagonal entries):

$$A = \begin{pmatrix} -1 & 1 & 2 & 1 \\ 1 & 2 & -2 & 3 \\ 3 & 2 & -3 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} -1 & 1 & 2 & 1 \\ 0 & 3 & 0 & 4 \\ 0 & 5 & 3 & 4 \\ 0 & 2 & 3 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} -1 & 1 & 2 & 1 \\ 0 & 1 & -3 & 2 \\ 0 & 5 & 3 & 4 \\ 0 & 2 & 3 & 2 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} -1 & 1 & 2 & 1 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 18 & -6 \\ 0 & 0 & 9 & -2 \end{pmatrix} \rightarrow \begin{pmatrix} -1 & 1 & 2 & 1 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 9 & -4 \\ 0 & 0 & 9 & -2 \end{pmatrix} \rightarrow \begin{pmatrix} -1 & 1 & 2 & 1 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 9 & -4 \\ 0 & 0 & 0 & 2 \end{pmatrix} = R$$

So $\det(A) = \det(R) = (-1)(1)(9)(2) = -18$.