Math 314/814, Section 4

Quiz number 7 Solutions

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

Find bases for the column space Col(A) and nullspace Nul(A) of the matrix

$$A = \begin{pmatrix} 3 & 1 & -5 \\ 1 & 2 & 5 \\ 3 & 4 & 7 \end{pmatrix}.$$

Since we can find each of these bases from a row reduction of A, we row reduce!

$$A = \begin{pmatrix} 3 & 1 & -5 \\ 1 & 2 & 5 \\ 3 & 4 & 7 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 5 \\ 3 & 1 & -5 \\ 3 & 4 & 7 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 5 \\ 0 & -5 & -20 \\ 0 & -2 & -8 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 2 & 5 \\ 0 & 1 & 4 \\ 0 & -2 & -8 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 5 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & -3 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{pmatrix}$$

Since the pivots are in the first and second columns, those column vectors of A,

$$\begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$$

are a basis for Col(A). Solving $A\vec{x} = \vec{0}$ gives us our other basis:

$$\begin{pmatrix} 1 & 0 & -3 & | & 0 \\ 0 & 1 & 4 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$ gives $x - 3z = 0$, $y + 4z = 0$, so

$x = 3z$, $y = -4z$, and $z = z$, and so $\begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$ is a basis for Nul(A).