

Quiz number 1 Solution

Find a solution to the system of equations

$$\begin{aligned} 3x - 2y + z &= 6 \\ x - y + 3z &= 7 \\ -4x + 5y - z &= -5 \end{aligned}$$

Solution: There are any number of ways to solve this. Here is one.

Rewriting this in matrix form, and applying row reduction steps:

$$\text{Start: } \left(\begin{array}{ccc|c} 3 & -2 & 1 & 6 \\ 1 & -1 & 3 & 7 \\ -4 & 5 & -1 & -5 \end{array} \right)$$

$$\text{swap rows: } \left(\begin{array}{ccc|c} 1 & -1 & 3 & 7 \\ 3 & -2 & 1 & 6 \\ -4 & 5 & -1 & -5 \end{array} \right)$$

$$\text{add multiples of top row: } \left(\begin{array}{ccc|c} 1 & -1 & 3 & 7 \\ 0 & 1 & -8 & -15 \\ -4 & 5 & -1 & -5 \end{array} \right) \quad \left(\begin{array}{ccc|c} 1 & -1 & 3 & 7 \\ 0 & 1 & -8 & -15 \\ 0 & 1 & 11 & 23 \end{array} \right)$$

$$\text{add multiple of middle row: } \left(\begin{array}{ccc|c} 1 & -1 & 3 & 7 \\ 0 & 1 & -8 & -15 \\ 0 & 0 & 19 & 38 \end{array} \right)$$

$$\text{rescale bottom row: } \left(\begin{array}{ccc|c} 1 & -1 & 3 & 7 \\ 0 & 1 & -8 & -15 \\ 0 & 0 & 1 & 2 \end{array} \right)$$

Then we can either backsolve: $z = 2$, $y - 8z = y - 16 = -15$, so $y = 1$,
and $x - y + 3z = x - 1 + 6 = x - 5 = 7$, so $x = 2$, or continue row reduction:

$$\text{add multiples of bottom row: } \left(\begin{array}{ccc|c} 1 & -1 & 3 & 7 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array} \right) \quad \left(\begin{array}{ccc|c} 1 & -1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array} \right)$$

$$\text{add multiple of middle row: } \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array} \right)$$

so $x = 2$, $y = 1$, $z = 2$.

So $x = 2$, $y = 1$, $z = 2$ is a solution to our original system of equations.