Math 314/814, Section 5

Quiz number 4 Solutions

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

Find the inverse of the matrix

$$A = \begin{pmatrix} 1 & -2 & 0 \\ 3 & -2 & 1 \\ -2 & 7 & 1 \end{pmatrix},$$

and use this to find solutions to the equation $A\vec{x} = \vec{b}$ for

$$\vec{b} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} \text{ and } \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}.$$

We build the superaugmented matrix $\left(\begin{array}{ccc|ccc} 1 & -2 & 0 & 1 & 0 & 0 \\ 3 & -2 & 1 & 0 & 1 & 0 \\ -2 & 7 & 1 & 0 & 0 & 1 \end{array}\right)$

and row reduce:

$$\left(\begin{array}{ccc|ccc} 1 & -2 & 0 & 1 & 0 & 0 \\ 0 & 4 & 1 & -3 & 1 & 0 \\ 0 & 3 & 1 & 2 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & -2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1/4 & -3/4 & 1/4 & 0 \\ 0 & 3 & 1 & 2 & 0 & 1 \end{array}\right) \to$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1/2 & -1/2 & 1/2 & 0 \\ 0 & 1 & 1/4 & -3/4 & 1/4 & 0 \\ 0 & 0 & 1/4 & 17/4 & -3/4 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 0 & 1/2 & -1/2 & 1/2 & 0 \\ 0 & 1 & 1/4 & -3/4 & 1/4 & 0 \\ 0 & 0 & 1 & 17 & -3 & 4 \end{array}\right) \to$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -9 & 2 & -2 \\ 0 & 1 & 0 & -5 & 1 & -1 \\ 0 & 0 & 1 & 17 & -3 & 4 \end{array}\right)$$

So the inverse of A is $B = \begin{pmatrix} -9 & 2 & -2 \\ -5 & 1 & -1 \\ 17 & -3 & 4 \end{pmatrix}$. So

$$A\vec{x} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} \text{ has solution } \begin{pmatrix} -9 & 2 & -2 \\ -5 & 1 & -1 \\ 17 & -3 & 4 \end{pmatrix} \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} -7 \\ -4 \\ 16 \end{bmatrix}, \text{ and}$$

$$A\vec{x} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} \text{ has solution } \begin{pmatrix} -9 & 2 & -2 \\ -5 & 1 & -1 \\ 17 & -3 & 4 \end{pmatrix} \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \\ 9 \end{bmatrix}.$$