

Quiz number 5 Solutions

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

Find bases for the column space of the matrix $A = \begin{pmatrix} 2 & 3 & 4 \\ 5 & 7 & 9 \\ 1 & 4 & 7 \end{pmatrix}$, by

(a) row reducing the matrix A ,

(b) row reducing the transpose A^T of the matrix A .

$$\begin{aligned} A &= \begin{pmatrix} 2 & 3 & 4 \\ 5 & 7 & 9 \\ 1 & 4 & 7 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 4 & 7 \\ 2 & 3 & 4 \\ 1 & 4 & 7 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 4 & 7 \\ 0 & -5 & -10 \\ 0 & -13 & -26 \end{pmatrix} \\ &\rightarrow \begin{pmatrix} 1 & 4 & 7 \\ 0 & 1 & 2 \\ 0 & -13 & -26 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 4 & 7 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \end{aligned}$$

The REF has pivots in the first two columns, so the first two columns of A form a basis for $\text{col}(A)$.

$$\text{basis} = \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 7 \\ 4 \end{bmatrix}$$

$$\begin{aligned} A^T &= \begin{pmatrix} 2 & 5 & 1 \\ 3 & 7 & 4 \\ 4 & 8 & 7 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 5/2 & 1/2 \\ 3 & 7 & 4 \\ 4 & 8 & 7 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 5/2 & 1/2 \\ 0 & -1/2 & 5/2 \\ 0 & -1 & 5 \end{pmatrix} \\ &\rightarrow \begin{pmatrix} 1 & 5/2 & 1/2 \\ 0 & 1 & -5 \\ 0 & -1 & 5 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 13 \\ 0 & 1 & -5 \\ 0 & 0 & 0 \end{pmatrix} \end{aligned}$$

The non-zero rows, $[1 \ 0 \ 13]$ and $[0 \ 1 \ -5]$, form a basis for $\text{row}(A^T)$. So their transposes,

$$\begin{bmatrix} 1 \\ 0 \\ 13 \end{bmatrix} \text{ and } \begin{bmatrix} 0 \\ 1 \\ -5 \end{bmatrix}, \text{ form a basis for } \text{col}(A).$$