Math 314/814, Section 5

Quiz number 6 Solutions

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

1. Explain why the function $f : \mathbf{R}^2 \to \mathbf{R}^2$ given by
$$f(x,y) = (x-y, x^2+y^2)$$
is **not** a linear transformation.

   $f((x,y)+(a,b)) = f(x+a, y+b) = ((x+a)-(y+b), (x+a)^2+(y+b)^2)$
   $= ((x-y)+(a-b), (x^2+y^2)+(a^2+b^2)+(2ax+2yb))$
   $= (x-y, x^2+y^2)+(a-b, a^2+b^2)+(0, 2ax+2yb) = f(x,y)+f(a,b)+(0, 2ax+2yb)$, which does **not** equal $f(x,y)+f(a,b)$ in general, e.g. for $x=y=a=b=1$.

   Alternatively, $f(c(x,y)) = f(cx,cy) = (cx-cy, (cx)^2+(cy)^2) = (cx-cy, c^2x^2+c^2y^2)$, while
   $cf(x,y) = c(x-y, x^2+y^2) = (cx-cy, cx^2+cy^2)$, which does **not** equal $f(c(x,y))$ in general, since
   $c^2x^2+c^2y^2 \neq cx^2+cy^2$ in general, e.g. for $x=y=1$ and $c=672$.

2. Find the matrix A so that $T = T_A$, where $T : \mathbf{R}^2 \to \mathbf{R}^2$ is the linear transformation which given a vector $[x \ \ y]^T$ returns the vector $[y \ \ x]^T$. Geometrically, what does this transformation do?

   $$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} 0\cdot x + 1\cdot y \\ 1\cdot x + 0\cdot y \end{bmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

   So $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

   Geometrically, the transformation sends $(0,1)$ to $(1,0)$ and sends $(0,1)$ to $(1,0)$; in general, the transformation swaps the x- and y-coordinates, which can be accomplished by reflecting the vector across the line $y=x$. So T is reflection across the line $y=x$.