Math 314/814, Section 5

Quiz number 7 solutions

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

Find the eigenvalues of the matrix

$$A = \begin{pmatrix} 2 & 3 \\ 4 & 3 \end{pmatrix}$$

and, for each eigenvalue, find a basis for its eigenspace.

To find the eigenvalues, we solve the equation

$$(2-\lambda)(3-\lambda)-(3)(4) = 0 = \lambda^2 - 5\lambda + 6 - 12 = \lambda^2 - 5\lambda - 6 = (\lambda-6)(\lambda+1),$$

so $\lambda = 6$ or $\lambda = -1$. These are our eigenvalues.

To find bases for the eigenspaces we find bases for the nullspaces of $A-6I$ and $A-(-1)I = A+I$, by row reducing:

$$A - 6I = \begin{pmatrix} 2-6 & 3 \\ 4 & 3-6 \end{pmatrix} = \begin{pmatrix} -4 & 3 \\ 4 & -3 \end{pmatrix} \to \begin{pmatrix} 1 & -3/4 \\ 4 & -3 \end{pmatrix} \to \begin{pmatrix} 1 & -3/4 \\ 0 & 0 \end{pmatrix}$$

so the second variable is free and we have $x - (3/4)y = 0$ so $x = (3/4)y$.

So E_6 has basis $\begin{bmatrix} 3/4 \\ 1 \end{bmatrix}$ (or any non-zero multiple of this).

$$A + I = \begin{pmatrix} 2+1 & 3 \\ 4 & +1 \end{pmatrix} = \begin{pmatrix} 3 & 3 \\ 4 & 4 \end{pmatrix} \to \begin{pmatrix} 1 & 1 \\ 4 & 4 \end{pmatrix} \to \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$$

so the second variable is again free and we have $x + y = 0$ so $x = -y$.

So E_{-1} has basis $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ (or any non-zero multiple of this).

We can check these answers:

$$A\begin{bmatrix} 3/4 \\ 1 \end{bmatrix} = \begin{bmatrix} 2(3/4)+3(1) \\ 4(3/4)+3(1) \end{bmatrix} = \begin{bmatrix} 9/2 \\ 6 \end{bmatrix} = 6\begin{bmatrix} 3/4 \\ 1 \end{bmatrix}, \text{ and}$$

$$A\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2(-1)+3(1) \\ 4(-1)+3(1) \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} = (-1)\begin{bmatrix} -1 \\ 1 \end{bmatrix}, \text{ as desired.}$$