

Quiz number 8 Solutions

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

For what values of x is the matrix

$$A = \begin{pmatrix} x & 1 & 3 \\ 3 & 1 & x \\ 0 & -1 & x \end{pmatrix}$$

invertible?

A is invertible precisely when its determinant is non-zero. So we compute

$$\begin{aligned} \det(A) &= x \cdot \det \begin{pmatrix} 1 & x \\ -1 & x \end{pmatrix} - 3 \cdot \det \begin{pmatrix} 1 & 3 \\ -1 & x \end{pmatrix} + 0 \cdot \det \begin{pmatrix} 1 & 3 \\ 1 & x \end{pmatrix} \\ &= x(x - (-x)) - 3(x - (-3)) + 0(x - 3) = 2x^2 - 3x - 9. \end{aligned}$$

To find out where it is non-zero, we find out where it is zero:

$$\begin{aligned} 2x^2 - 3x - 9 = 0 \text{ for } x &= \frac{3 \pm \sqrt{9 + 4 \cdot 2 \cdot 9}}{2 \cdot 2} = \frac{3 \pm \sqrt{81}}{4}, \\ \text{so } x &= (3 + 9)/4 = 3 \text{ or } x = (3 - 9)/4 = -3/2 \end{aligned}$$

So, for $x \neq -3/2, 3$, A is invertible.