Math 314/814, Section 5

Quiz number 9 Solutions

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

For the matrix

$$A = \begin{pmatrix} 8 & -18 \\ 3 & -7 \end{pmatrix},$$

Find a matrix P so that $P^{-1}AP = D$ is a diagonal matrix; what is D?

First, compute eigenvalues:

$\det(A - \lambda I) = \det\begin{pmatrix} 8-\lambda & -18 \\ 3 & -7-\lambda \end{pmatrix} = (8-\lambda)(-7-\lambda) - (-18)(3)$

$= (-56 - 8\lambda + 7\lambda + \lambda^2) + 54 = \lambda^2 - \lambda + 2 = (\lambda - 2)(\lambda + 1)$

so the eigenvalues of A are 2 and -1. The eigenbases:

$\lambda = 2$: $(A - 2I) = \begin{pmatrix} 8-2 & -18 \\ 3 & -7-2 \end{pmatrix} = \begin{pmatrix} 6 & -18 \\ 3 & -9 \end{pmatrix} \to \begin{pmatrix} 1 & -3 \\ 3 & -9 \end{pmatrix} \to \begin{pmatrix} 1 & -3 \\ 0 & -0 \end{pmatrix}$,

so y is free and $x - 3y = 0$, so $x = 3y$ so $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ is a basis for E_2.

$\lambda = 2$: $(A - (-1)I) = \begin{pmatrix} 8-(-1) & -18 \\ 3 & -7-(-1) \end{pmatrix} = \begin{pmatrix} 9 & -18 \\ 3 & -6 \end{pmatrix} \to$
$\begin{pmatrix} 1 & -2 \\ 3 & -6 \end{pmatrix} \to \begin{pmatrix} 1 & -2 \\ 0 & 0 \end{pmatrix}$,

so y is free and $x - 2y = 0$, so $x = 2y$, so $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ is a basis for E_{-1}.

Then we have, for $P = \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$, $AP = PD$ for $D = \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}$,

so $P = \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$ and $D = \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}$.

Note that any other non-zero multiple of the eigenvectors could be chosen to build the matrix P, and the columns could be written in either order. The corresponding placement of the eigenvalues of A in D would need to be written in the other order, however.