Math 314/814

Things we know how to do

Solve a system of equations $A\vec{x} = \vec{b}$ using Gaussian elimination.

Show that a linear system has no solutions, one solution, many solutions (check for consistency, count free variables).

Compute the rank and nullity of a matrix.

Show that a collection of vectors in $\mathbb{R}^n$ span $\mathbb{R}^n$ (pivot in every row), show they are linearly independent (no free variables).

Compute the net flow through a network, by monitoring (i.e., knowing the value at) some of the edges.

Write an (invertible) matrix as a product of elementary matrices (by keeping track of the row operations in reducing it to the identity matrix).

Compute the inverse of a matrix A (using a super-augmented matrix).

Determine if a collection of vectors form a subspace (check closure under addition, scalar multiplication).

Interpret linear systems in terms of column and nullspaces (column = who has solutions, null = how many).

Find bases for column space, row space, nullspace (row reduce!).

Start with linearly independent vectors in a subpace, extend to a basis (add a basis at the end, then row reduce, keep the columns corresponding to pivots).

Start with a spanning set for a subspace, choose a basis (row reduce, keep columns corresponding to pivots).

Compute the matrix for a linear transformation; compute the image of a vector under a transformation.

Compute the determinant of a matrix (by row reduction, by expanding along row/column).

Compute the solution to a system of equations $A\vec{x} = \vec{b}$ with A invertible (by inverting! or Cramer's rule).

Compute the characteristic polynomial of a matrix.

Compute the eigenvalues and bases for eigenspaces for a matrix.

Diagnonalize a matrix, or show that it cannot be done (geometric vs. algebraic multiplicity). Use diagonalization to compute "high" powers of a matrix.

Show two matrices *aren't* similar (by showing they have different eigenvalues, or characteristic polynomials, or geometric multiplicities).

Build a basis for the orthogonal complement of a subspace (described as span? $(\text{col}(A))^\perp = \text{null}(A^T)$. described as nullspace? $(\text{null}(A))^\perp = \text{row}(A)$.)

Use $(\text{col}(A))^\perp = \text{null}(A^T)$ to build a test for consistency of a system $A\vec{x} = \vec{b}$ ($\vec{b}$ must be $\perp$ a basis for $\text{null}(A^T)$).

Build an orthogonal (orthonormal) basis for a subspace (start with a basis, and apply Gram-Schmidt).

Compute the orthogonal projection of a vector to a subspace (build an orthogonal basis, and sum the projections onto each basis vector [or see below!]).

Decompose a vector $\vec{v}$ into the sum of a vector $\vec{w} \in W$ and $\vec{w}' \in W^\perp$.

Find the vector in $\text{col}(A)$ closest to $\vec{b}$ (solve $A^TA\vec{x} = A^T\vec{b}$, take $A\vec{x}$). [This is the same as taking the orthogonal projection of $\vec{b}$ onto $\text{col}(A)$.]

Note! If we choose a basis for $\text{col}(A)$ and assemble them into a matrix (which we will still call A), then A^TA is invertible. So $A^TA\vec{x} = A^T\vec{b}$ has solution $\vec{x} = (A^TA)^{-1}A^T\vec{b}$, so $A\vec{x} = A(A^TA)^{-1}A^T\vec{b}$. So $A(A^TA)^{-1}A^T$ is the matrix for the $\perp$ projection onto $\text{col}(A)$, when A has a pivot in every column.

Find the line which best fits a collection of data points.

Orthogonally diagonalize a symmetric matrix!