Name: Solutions

Math 314/814

Quiz number 9

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

The matrix

$$A = \begin{pmatrix} 0 & -2 & 1 \\ 1 & 3 & -5 \\ 1 & 1 & -3 \end{pmatrix}$$

has characteristic polynomial $\chi_A(t) = t^3 - 3t - 2$. (You need not verify this.) Find the eignevlues for A and, for each eigenvalue, a basis for the corresponding eigenspace.

$t^3-3t-2=0$? Try $t=\pm 1, \pm 2$. $1^3-3-2=-4\neq 0$, $-1+3-2=0$ ✓ $t=-1$

$t^3-3t-2=(t+1)(t^2-t-2)=(t+1)(t-2)(t+1)=(t+1)^2(t-2)=0$

$\Leftrightarrow t=-1, t=2$ ← eigenvalues

$t=-1$: $A-(-1)I = A+I = \begin{pmatrix} 1 & -2 & 1 \\ 1 & 4 & -5 \\ 1 & 1 & -2 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & -2 \\ 1 & -2 & 1 \\ 1 & 4 & -5 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & -2 \\ 0 & -3 & 3 \\ 0 & 3 & -3 \end{pmatrix}$

$\to \begin{pmatrix} 1 & 1 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$ ↑ free var $\quad x-z=0$, $y-z=0$ $\quad x=z$, $y=z$ $\quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} z \\ z \\ z \end{pmatrix} = z\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ ← basis

$t=2$: $A-2I = \begin{pmatrix} -2 & -2 & 1 \\ 1 & 1 & -5 \\ 1 & 1 & -5 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & -5 \\ -2 & -2 & 1 \\ 1 & 1 & -5 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & -5 \\ 0 & 0 & -9 \\ 0 & 0 & 0 \end{pmatrix}$

$\to \begin{pmatrix} 1 & 1 & -5 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ ↑ free var $\quad x+y=0$, $z=0$ $\quad x=-y$ $\quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -y \\ y \\ 0 \end{pmatrix} = y\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ ← basis

So: eigenvalues are $\lambda=-1$, $\lambda=2$:

$E_{-1}(A)$ has basis $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$E_2(A)$ has basis $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$