Math 314/814, Section 6

Quiz number 10 Solutions

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

Find the eigenvalues of the matrix $A = \begin{pmatrix} -1 & 3 \\ 2 & 4 \end{pmatrix}$, and find bases for each of the associated eigenspaces.

First we find the eigenvalues, by factoring the characteristic polynomial:

$$\chi_A(\lambda) = \det \begin{pmatrix} -1-\lambda & 3 \\ 2 & 4-\lambda \end{pmatrix}$$
$$= (-1-\lambda)(4-\lambda) - (3)(2) = -4 - 4\lambda + \lambda + \lambda^2 - 6 = \lambda^2 - 3\lambda - 10 = (\lambda+2)(\lambda-5)$$

so the eigenvalues are $\lambda = -2$ and $\lambda = 5$.

Then we compute bases for the corresponding nullspaces:

$\lambda = -2$:

$$A - (-2)I_2 = A + 2I_2 = \begin{pmatrix} -1+2 & 3 \\ 2 & 4+2 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 3 \\ 2 & 6 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 3 \\ 0 & 0 \end{pmatrix}$$

so a null vector has $x + 3y = 0$, so $x = -3y$, so $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$ is a basis for the (-2)-eigenspace.

$\lambda = 5$:

$$A-(5)I_2 = \begin{pmatrix} -1-5 & 3 \\ 2 & 4-5 \end{pmatrix} = \begin{pmatrix} -6 & 3 \\ 2 & -1 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & -1 \\ -6 & 3 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & -1 \\ 0 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & -1/2 \\ 0 & 0 \end{pmatrix}$$

so a null vctor has $x - (1/2)y = 0$, so $x = (1/2)y$, so $\begin{pmatrix} 1/2 \\ 1 \end{pmatrix}$ (or $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$, if we like integers) is a basis for the 5-eigenspace.

Note: we can check that our answers are correct, by matrix multiplication:

$$A\begin{pmatrix} -3 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 & 3 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} -3 \\ 1 \end{pmatrix} = \begin{pmatrix} (-1)(-3)+(3)(1) \\ (2)(-3)+(4)(1) \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \end{pmatrix} = (-2)\begin{pmatrix} -3 \\ 1 \end{pmatrix}$$

$$A\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 & 3 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} (-1)(1)+(3)(2) \\ (2)(1)+(4)(2) \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \end{pmatrix} = (5)\begin{pmatrix} 1 \\ 2 \end{pmatrix}$$