

Quiz number 1 Solution

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

Use any method to find a solution to the system of equations

$$\begin{aligned} 2x + y + 2z &= 9 \\ x - y + 3z &= 7 \\ -2x + 5y + z &= 3 \end{aligned}$$

Solution: There are any number of ways to solve this. Here is one.

Rewriting this in matrix form, and applying our rules for changing the SLE without changing the solutions (i.e., row reduction):

$$\text{Start: } \left(\begin{array}{ccc|c} 2 & 1 & 2 & 9 \\ 1 & -1 & 3 & 7 \\ -2 & 5 & 1 & 3 \end{array} \right) \quad \text{Switch rows: } \left(\begin{array}{ccc|c} 1 & -1 & 3 & 7 \\ 2 & 1 & 2 & 9 \\ -2 & 5 & 1 & 3 \end{array} \right)$$

$$\text{Add multiples of first row: } \left(\begin{array}{ccc|c} 1 & -1 & 3 & 7 \\ 0 & 3 & -4 & -5 \\ -2 & 5 & 1 & 3 \end{array} \right) \quad \left(\begin{array}{ccc|c} 1 & -1 & 3 & 7 \\ 0 & 3 & -4 & -5 \\ 0 & 3 & 7 & 17 \end{array} \right)$$

Subtract second row from third (i.e., add (-1) times second row to third):

$$\left(\begin{array}{ccc|c} 1 & -1 & 3 & 7 \\ 0 & 3 & -4 & -5 \\ 0 & 0 & 11 & 22 \end{array} \right)$$

Divide third row by 11 (i.e., multiply by 1/11):

$$\left(\begin{array}{ccc|c} 1 & -1 & 3 & 7 \\ 0 & 3 & -4 & -5 \\ 0 & 0 & 1 & 2 \end{array} \right)$$

Add multiples of third row:

$$\left(\begin{array}{ccc|c} 1 & -1 & 3 & 7 \\ 0 & 3 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{array} \right) \quad \left(\begin{array}{ccc|c} 1 & -1 & 0 & 1 \\ 0 & 3 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{array} \right)$$

Multiply second row by 1/3, then add second row to first:

$$\left(\begin{array}{ccc|c} 1 & -1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array} \right) \quad \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array} \right)$$

which “rehydrates” to our solution: $x = 2, y = 1, z = 2$.

We can check that this *is* a solution:

$$2(2) + (1) + 2(2) = 4 + 1 + 4 = 9$$

$$(2) - (1) + 3(2) = 2 - 1 + 6 = 7$$

$$-2(2) + 5(1) + (2) = -4 + 5 + 2 = 3$$