Quiz number 2 Solution

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

Show that the vector $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$ is **not** in the span of the vectors $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}$.

(Show that it <u>cannot</u> be expressed as a linear combination....)

We are being asked to show that the equation

$$
x \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} + z \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}
$$

has no solution, that is, the SLE $\begin{pmatrix} 1 & 3 & 1 & | & 2 \\ 2 & 2 & 4 & | & 1 \\ 3 & 1 & 7 & | & 2 \end{pmatrix}$ is inconsistent.

We can determine this by row reduction:

$$
\begin{pmatrix} 1 & 3 & 1 & | & 2 \\ 2 & 2 & 4 & | & 1 \\ 3 & 1 & 7 & | & 2 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 3 & 1 & | & 2 \\ 0 & -4 & 2 & | & -3 \\ 3 & 1 & 7 & | & 2 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 3 & 1 & | & 2 \\ 0 & -4 & 2 & | & -3 \\ 0 & -8 & 4 & | & -4 \end{pmatrix}
$$

$$
\longrightarrow \begin{pmatrix} 1 & 3 & 1 & | & 2 \\ 0 & 1 & -1/2 & | & 3/4 \\ 0 & -8 & 4 & | & -4 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 3 & 1 & | & 2 \\ 0 & 1 & -1/2 & | & 3/4 \\ 0 & 0 & 0 & | & 2 \end{pmatrix}
$$

The last row of the last augmented matrix rehydrates to "$0 = 2$", which is never true, and so the system is inconsistent. So there are no values of x, y, and z which will make the original equation true;

$\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$ cannot be written as a linear combination of $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}$,

and so does not lie in their span.