Math 314 Section 5

Quiz number 4 Solutions

$T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation with $T\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\2\end{pmatrix}$ and $T\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}3\\-1\end{pmatrix}$.
What is $T\begin{pmatrix}2\\-3\end{pmatrix}$?

Since T is linear, we have

$$\begin{aligned} T\begin{pmatrix}2\\-3\end{pmatrix} &= T(\begin{pmatrix}2\\0\end{pmatrix} + \begin{pmatrix}0\\-3\end{pmatrix}) = T\begin{pmatrix}2\\0\end{pmatrix} + T\begin{pmatrix}0\\-3\end{pmatrix} = T(2\begin{pmatrix}1\\0\end{pmatrix}) + T(-3\begin{pmatrix}0\\1\end{pmatrix}) \\ &= 2T\begin{pmatrix}1\\0\end{pmatrix} + (-3)T\begin{pmatrix}0\\1\end{pmatrix} = 2\begin{pmatrix}1\\2\end{pmatrix} + (-3)T\begin{pmatrix}3\\-1\end{pmatrix} = \begin{pmatrix}2\cdot 1 + (-3)\cdot 3\\ 2\cdot 2 + (-3)\cdot(-1)\end{pmatrix} \\ &= \begin{pmatrix}-7\\7\end{pmatrix}. \end{aligned}$$

$S : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation with $S\begin{pmatrix}3\\2\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$ and $S\begin{pmatrix}2\\-1\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$.
Find a vector $\begin{pmatrix}x\\y\end{pmatrix}$ so that $S\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}4\\5\end{pmatrix}$.

Since $\begin{pmatrix}4\\5\end{pmatrix} = 4\begin{pmatrix}1\\0\end{pmatrix} + 5\begin{pmatrix}0\\1\end{pmatrix}$, we have, since S is linear,

$$\begin{aligned} S(4\begin{pmatrix}3\\2\end{pmatrix} + 5\begin{pmatrix}2\\-1\end{pmatrix}) &= S(4\begin{pmatrix}3\\2\end{pmatrix}) + S(5\begin{pmatrix}2\\-1\end{pmatrix}) = 4S\begin{pmatrix}3\\2\end{pmatrix} + 5S\begin{pmatrix}2\\-1\end{pmatrix} \\ &= 4\begin{pmatrix}1\\0\end{pmatrix} + 5\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}4\\5\end{pmatrix}. \end{aligned}$$

So $\begin{pmatrix}x\\y\end{pmatrix} = 4\begin{pmatrix}3\\2\end{pmatrix} + 5\begin{pmatrix}2\\-1\end{pmatrix} = \begin{pmatrix}4\cdot 3 + 5\cdot 2\\ 4\cdot 2 + 5\cdot(-1)\end{pmatrix} = \begin{pmatrix}22\\3\end{pmatrix}$ is a vector with
$S\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}4\\5\end{pmatrix}$.