Math 314/814, Section 5

Quiz number 5 Solution

Use the super-augmented matrix to find the inverse of the matrix

$$A = \begin{pmatrix} 1 & 3 & 2 \\ -1 & 1 & 1 \\ 3 & 2 & 1 \end{pmatrix} .$$

Building the super-augmented matrix and row reducing it, we find:

$$(A|I_3) = \left(\begin{array}{ccc|ccc} 1 & 3 & 2 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 & 0 & 1 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|ccc} 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 4 & 3 & 1 & 1 & 0 \\ 3 & 2 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{ccc|ccc} 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 4 & 3 & 1 & 1 & 0 \\ 0 & -7 & -5 & -3 & 0 & 1 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|ccc} 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3/4 & 1/4 & 1/4 & 0 \\ 0 & -7 & -5 & -3 & 0 & 1 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{ccc|ccc} 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3/4 & 1/4 & 1/4 & 0 \\ 0 & 0 & 1/4 & -5/4 & 7/4 & 1 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|ccc} 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3/4 & 1/4 & 1/4 & 0 \\ 0 & 0 & 1 & -5 & 7 & 4 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{ccc|ccc} 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 4 & -5 & -3 \\ 0 & 0 & 1 & -5 & 7 & 4 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|ccc} 1 & 3 & 0 & 11 & -14 & -8 \\ 0 & 1 & 0 & 4 & -5 & -3 \\ 0 & 0 & 1 & -5 & 7 & 4 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 1 \\ 0 & 1 & 0 & 4 & -5 & -3 \\ 0 & 0 & 1 & -5 & 7 & 4 \end{array}\right) = (I_3|B) .$$

Then we know that $BA = I_3$, and so $A^{-1} = B = \begin{pmatrix} -1 & 1 & 1 \\ 4 & -5 & -3 \\ -5 & 7 & 4 \end{pmatrix}$.