Quiz number 7 Solutions

Use a superaugmented matrix to find a/the matrix B whose nullspace is equal to the column space of the matrix

$$A = \begin{pmatrix} 3 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & -5 & 2 \end{pmatrix} \text{ , and use this to decide if the linear system } A\vec{x} = \vec{b}$$

is consistent, for each of the vectors $\vec{b} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$.

We row reduce:

$$\left(\begin{array}{ccc|ccc} 3 & 1 & 2 & 1 & 0 & 0 \\ 2 & 2 & 1 & 0 & 1 & 0 \\ 1 & -5 & 2 & 0 & 0 & 1 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|ccc} 1 & -5 & 2 & 0 & 0 & 1 \\ 3 & 1 & 2 & 1 & 0 & 0 \\ 2 & 2 & 1 & 0 & 1 & 0 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{ccc|ccc} 1 & -5 & 2 & 0 & 0 & 1 \\ 0 & 16 & -4 & 1 & 0 & -3 \\ 2 & 2 & 1 & 0 & 1 & 0 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|ccc} 1 & -5 & 2 & 0 & 0 & 1 \\ 0 & 16 & -4 & 1 & 0 & -3 \\ 0 & 12 & -3 & 0 & 1 & -2 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{ccc|ccc} 1 & -5 & 2 & 0 & 0 & 1 \\ 0 & 12 & -3 & 0 & 1 & -2 \\ 0 & 16 & -4 & 1 & 0 & -3 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|ccc} 1 & -5 & 2 & 0 & 0 & 1 \\ 0 & 1 & -1/4 & 0 & 1/12 & -1/6 \\ 0 & 16 & -4 & 1 & 0 & -3 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{ccc|ccc} 1 & -5 & 2 & 0 & 0 & 1 \\ 0 & 1 & -1/4 & 0 & 1/12 & -1/6 \\ 0 & 0 & 0 & 1 & -16/12 & -3+16/6 \end{array}\right)$$

$$= \left(\begin{array}{ccc|ccc} 1 & -5 & 2 & 0 & 0 & 1 \\ 0 & 1 & -1/4 & 0 & 1/12 & -1/6 \\ 0 & 0 & 0 & 1 & -4/3 & -1/3 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|ccc} 1 & -5 & 2 & 0 & 0 & 1 \\ 0 & 1 & -1/4 & 0 & 1/12 & -1/6 \\ 0 & 0 & 0 & 3 & -4 & -1 \end{array}\right)$$

So for $A\vec{x} = \vec{b}$ to be consistent, we need $B\vec{b} = \vec{0}$, where $B = \begin{pmatrix} 3 & -4 & -1 \end{pmatrix}$.

So we compute:

$$B \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = [3 + (-4) - (-1)] = [3 - 4 + 1] = [0] = \vec{0} \text{ , so this system is consistent.}$$

$$B \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = [3 - (-4) + (-1)] = [3 + 4 - 1] = [6] \neq \vec{0} \text{ , so this system is inconsistent.}$$

$$B \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = [-(3) + (-4) + (-1)] = [-3 - 4 - 1] = [-8] \neq \vec{0}, \text{ so this system is inconsistent.}$$