Math 314/814, Section 5

Quiz number 9 Solutions

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

By using the <u>transpose</u> A^T of the matrix $A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & -1 & 4 \\ 1 & 2 & 1 \end{pmatrix}$, find a basis for the column space of A, and use that basis to decide which of the systems of equations $A\vec{x} = \vec{b}$ are consistent, for $\vec{b}$ equal to

$\begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$, $\begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix}$, and $\begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$.

Treating the column space of A as the row space of A^T, we can find a basis for $Col(A)$ by row reducing A^T:

$$A^T = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 2 & 4 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 1 & 1 \\ 0 & -2 & 1 \\ 0 & 2 & -1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 1 & 1 \\ 0 & -2 & 1 \\ 0 & 0 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -1/2 \\ 0 & 0 & 0 \end{pmatrix}$$
$$\longrightarrow \begin{pmatrix} 1 & 0 & 3/2 \\ 0 & 1 & -1/2 \\ 0 & 0 & 0 \end{pmatrix}$$

This is in RREF, and so the (tranposes of the) non-$\vec{0}$ rows, $\begin{pmatrix} 1 \\ 0 \\ 3/2 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ -1/2 \end{pmatrix}$, are a basis for $\text{Row}(A^T) = \text{Col}(A)$.

But now our target vectors $\vec{b}$ give consistent systems precisely when they can be written

as a linear combination of our basis vectors, i.e. $\begin{pmatrix} a \\ b \\ c \end{pmatrix} = a\begin{pmatrix} 1 \\ 0 \\ 3/2 \end{pmatrix} + b\begin{pmatrix} 0 \\ 1 \\ -1/2 \end{pmatrix}$.

That is, $c = (3/2)a + (-1/2)b$. And we can test this!

$(3/2)(2) + (-1/2)(3) = 3/2 \neq 5$,so $A\vec{x} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$ is inconsistent;

$(3/2)(5) + (-1/2)(2) = 13/2 \neq 3$, so $A\vec{x} = \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix}$ is inconsistent;

$(3/2)(3) + (-1/2)(5) = 4/2 = 2$, so $A\vec{x} = \begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$ is consistent.