Math 314 Matrix Theory, old Exam 1 type problems

1. Use row reduction to find a solution to the system of equations:

$$\begin{array}{rcrcrcr} x & + & y & + & z & = & 6 \\ x & + & 2y & + & 2z & = & 11 \\ 3x & + & 2y & - & z & = & 4 \end{array}$$

2. (a) Use row reduction to decide if the system of linear equations represented by:

$$\left(\begin{array}{ccc|c} 2 & 1 & 1 & 6 \\ 1 & 2 & 1 & 5 \\ 4 & -1 & 1 & 3 \end{array}\right)$$

has a solution. If it does, does it have one or more than one solution?

(b) Answer the same question for the system

$$\left(\begin{array}{ccc|c} 2 & 1 & 1 & 6 \\ 1 & 2 & 1 & 7 \\ 4 & -1 & 1 & 4 \end{array}\right)$$

3. Use Gauss-Jordan elimination to find the inverse of the matrix A, where

$$\mathrm{A} = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 3 & 2 \\ 2 & 1 & 3 \end{pmatrix}$$

Use this to find the solution to the equation $\mathrm{A}x = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

4. Show that for every matrix A, $A^T A$ is always symmetric. (Hint: what is its transpose?!)

1. Use row reduction to find a solution to the following system of linear equations:

$$\begin{array}{rcrcrcr} x & + & y & - & z & = & 0 \\ 2x & + & 3y & + & 2z & = & 3 \\ -x & + & 5y & - & z & = & -8 \end{array}$$

2. Use row reduction to decide if the system of linear equations given by the augmented matrix:

$$(A|\mathbf{b}) = \left(\begin{array}{ccc|c} 1 & 1 & 1 & -1 \\ 1 & -2 & -1 & 5 \\ 1 & 7 & 5 & -1 \end{array}\right)$$

has a solution. If it does, does it have one or more than one solution? What is the rank of the matrix A ?

3. The company that markets Brand B tortilla chips plans to introduce their product into a town where everyone uses Brand A. Their research indicates that during each month, 1/2 of the brand A users will switch to their brand (the remainder will remain loyal), while 3/10ths of the users of Brand B will switch back to Brand A. What is the transition matrix that describes this Markov process? (5 pts.)

Based on this research, what fraction of the town's population will be Brand B users after 3 months? (15 pts. Hints: 36×7=252 ; 64×7=448 ; 64×3=192 .)

4. Use Gauss-Jordan elimination to find the inverse of the matrix A, where

$$\mathrm{A} = \begin{pmatrix} -1 & 1 & 2 \\ 1 & 2 & -1 \\ 2 & 0 & -3 \end{pmatrix}$$

(b) (5 pts.) Use your answer from part (a) to find the solution to the equation $\mathrm{A}x = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$

5. (a) Show that if A is an **invertible** $n \times n$ matrix with the property that

$$A^2 = A$$

then $A = I_n$. (8 pts.)

(b) Find an example of a (2×2 will do fine) square matrix A which is **not** invertible and has the property that $A^2 = A$. (7 pts.)