M340L Matrices, Old! Exam 2

Name:

Show all work.

4. (15 pts.) Let V=R^3 (3-dimensional Euclidean space) and let

$$W = \{(x, y, z) \in \mathbf{R^3} : 2x + 3y + 8z = 0\}.$$

Show that W is a **subspace** of V.

5. (10 pts.) Suppose L:$\mathbf{R^3} \to \mathbf{R^2}$ is a linear transformation, and suppose

$$L\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}2\\1\end{pmatrix}, \quad L\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}1\\2\end{pmatrix}, \quad \text{and} \quad L\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}-1\\3\end{pmatrix} \qquad \text{What is } L\begin{pmatrix}2\\-1\\3\end{pmatrix}?$$

2. The system of equations

$$\left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 2 & 3 & 4 & 0 & 1 & 0 & 0 \\ 3 & 3 & -1 & -6 & 0 & 0 & 1 & 0 \\ 1 & 1 & 2 & 3 & 0 & 0 & 0 & 1 \end{array}\right) \text{row-reduces to} \left(\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & 14 & -5 & -1 & 0 \\ 0 & 0 & 1 & 0 & -24 & 9 & 2 & 0 \\ 0 & 0 & 0 & 1 & 11 & -4 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right).$$

If we call the left-hand side of the first pair of matrices A, use this row-reduction information to find the dimensions and bases for the subspaces $\mathbf{R}(A)$, $\mathbf{N}(A)$, and $\mathbf{R}(A^T)$.

(5 pts. for each subspace.)

3. Do the vectors $\begin{pmatrix}1\\-1\\3\end{pmatrix}, \begin{pmatrix}2\\0\\2\end{pmatrix}, \begin{pmatrix}-1\\1\\1\end{pmatrix}$, and $\begin{pmatrix}1\\3\\5\end{pmatrix}$ span $\mathbf{R^3}$?

Are they linearly independent?

Can you find a subset of this collection of vectors which forms a basis for R^3?

(10 pts. for spanning, 10 pts. for lin indep, 5 pts. for basis.)

5. Find the orthogonal projection of the vector $\mathbf{x} = \begin{pmatrix}1\\2\\3\end{pmatrix}$ onto

the lines through $\begin{pmatrix}0\\0\\0\end{pmatrix}$ and the vectors (a) $\begin{pmatrix}1\\-1\\1\end{pmatrix}$, (b) $\begin{pmatrix}2\\2\\1\end{pmatrix}$, and (c) $\begin{pmatrix}1\\2\\-1\end{pmatrix}$

(5 pts. each).

1. (20 pts.) Find, using any method (other than psychic powers), the determinant of the matrix

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & -1 & 3 \\ 4 & 4 & 2 \end{pmatrix}$$

.

Is this matrix invertible?

2. (15 pts.) Explain why the set of vectors

$$W = \{(x, y, z) \mid x + y + 2z = 1\}$$

is **not** a subspace of $\mathbf{R}^3$.

3. (25 pts.) Show that the system of equations $Ax = b$, where

$$A = \begin{pmatrix} -1 & 1 \\ 1 & -2 \\ 0 & 1 \end{pmatrix} \text{ and } b = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$$

is **not** consistent. Find the least squares solution to this sytem, i.e., the value of Ax closest to b.

4.(20 pts.) For the matrix

$$A = \begin{pmatrix} 1 & 2 & 1 & 0 \\ 2 & 1 & 3 & 2 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

find bases for, and the dimensions of, the row, column, and null spaces of A.

5. (20 pts.) Find **all** of the solutions to the equation $Ax = b$, where

$$A = \begin{pmatrix} 1 & 2 & 2 & 1 \\ 2 & 4 & 3 & 3 \\ 1 & 2 & 1 & 2 \end{pmatrix} \text{ and } b = \begin{pmatrix} -2 \\ -2 \\ 0 \end{pmatrix}$$

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5. A friend of yours runs up to you and says 'Look I've found these three vectors v_1, v_2, v_3 in $\mathbf{R}^2$ that are linearly independent!' Explain how you know, without even looking at the vectors, that your friend is wrong (again).