Math 314 Matrix Theory Old! Exam 3

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

1. (25 pts.) Find the eigenvalues, and their associated eigenspaces, of the matrix

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & -1 & -1 \\ 1 & 3 & 2 \end{pmatrix}$$

.

Is this matrix diagonalizable?

2. (15 pts.) Show that if λ is an eigenvalue of a matrix A, then 3λ is an eigenvalue of the matrix $3A$. (Hint: **Don't** compute a determinant!)

3. (25 pts.) For the matrix

$$A = \begin{pmatrix} 1 & 3 \\ 4 & 5 \end{pmatrix}$$

find an (invertible!) matrix P and a diagonal matrix D so that $AP = PD$.

4.(20 pts.) (a) Verify that the vectors

$$u = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \text{ and } v = \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$$

are orthogonal, **if** we use the inner product $< u, v > = 3u_1v_1 + u_2v_2 + u_3v_3$ (instead of the ordinary one).

(b) Use the result of part (a) to write

$$\begin{pmatrix} 1 \\ 7 \\ 5 \end{pmatrix}$$

as a linear combination of u and v . (If you don't know what this is 'really' asking, you can do it the 'normal' way (for part credit).)

5. (15 pts.) Show why, for $u = (u_1, u_2, u_3) \in R^3$, the function

$$||u|| = |u_1| + |u_2| - |u_3|$$

is **not** a norm on R^3 .