Math 314 Matrix Theory
Old! Final Exam

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

1. (20 pts.) Find the inverse of the matrix $\begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 2 \\ 3 & 2 & 1 \end{pmatrix}$ and use this to find solutions to the systems of equations $Au = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ and $Av = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

2. (20 pts.) Find bases for, and the dimensions of, the row column , and nullspaces of the matrix

$$A = \begin{pmatrix} 1 & 2 & -1 \\ -1 & 0 & -3 \\ 3 & 1 & 7 \end{pmatrix}$$

3. (15 pts.) Find the value of Ax closest to b, where $A = \begin{pmatrix} 1 & 2 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $b = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$

4. (15 pts.) Find the determinant of the matrix $A = \begin{pmatrix} 1 & 2 & -7 \\ 1 & 0 & -3 \\ 3 & 1 & 3 \end{pmatrix}$. Based on this, find the determinants of the matrices $B = A^{-1}$, $C = A^T$, and $D = A^T A$. (Hint: you don't need to compute these matrices....)

5. (a) (5 pts.) Show that for the 2×2 diagonal matrix $D = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$ we have $(D - \lambda_1 I)(D - \lambda_2 I) = 0$.

(b) (10 pts.) Show that if the 2×2 matrix A has distinct eigenvalues $\lambda_1 \neq \lambda_2$, then $(A - \lambda_1 I)(A - \lambda_2 I) = 0$.

(Hint: I don't think this can be done by brute force; think 'similar to ...') (FYI: This is a special case of what is more generally known as the Cayley-Hamilton Theorem.)

6. (15 pts.)Use the Gram-Schmidt algorithm to find an orthogonal basis for the subspace of R^4 spanned by the vectors $\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 1 \\ 0 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 1 \\ 2 \\ -1 \end{pmatrix}$

7. (15 pts.) Find the orthogonal complement of the subspace W of R^4 spanned by the vectors

$$\begin{pmatrix} 1 \\ -1 \\ 0 \\ 2 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} -1 \\ 1 \\ 1 \\ 2 \end{pmatrix}$$