Math 325 Problem Set 3

Starred (*) problems are due Friday, September 14.

10. Show, by induction, that the (ordinary) triangle inequality extends to show that for any $n \geq 2$ we have

$$|\sum_{k=1}^{n} a_k| \leq \sum_{k=1}^{n} |a_k| \ .$$

(*) 11. Show that the <u>maximum</u> of two numbers $x, y \in \mathbb{R}$ can be computed by

$$\max(x, y) = \frac{x + y + |x - y|}{2} \ .$$

[That is, if $x \leq y$ then $\dfrac{x + y + |x - y|}{2} = y$, while if $y \leq x$ then it equals x.]

Find a similar formula which gives the minimum of x and y.

12. (Belding and Mitchell, p.35, #4) Under what conditions on $a, b, c \in \mathbb{R}$ can we conclude that $ab = cb$ implies that $a = c$? Prove your answer.

13. (Belding and Mitchell, p.3,5 #7) If we have a subset $A \subseteq \mathbb{R}$ of real numbers, then A satisfies all of the properties of a field so long as (a) 0 and 1 are elements of A and (b) whenever $a, b \in A$ we have $-a \in A$, $a^{-1} \in A$, $a + b \in A$ and $ab \in A$ (where these are the negative and inverse <u>in the real line</u>); that is, A is *closed* under negation, inverse, addition, and multiplication.

Use this to show that the set $\mathbb{Q}[\sqrt{11}] = \{a + b\sqrt{11} \ : \ a, b \in \mathbb{R}\}$ satisfies all of the properties of a field.

(*) 14. (Belding and Mitchell, p.36, #17) Use the triangle inequality to establish that for every $x, y \in \mathbb{R}$ we have

(*) (a) $|x| - |y| \leq |x - y|$

(*) (b) $|x| - |y| \leq |x + y|$

(*) (d) $\Big||x| - |y|\Big| \leq |x - y|$

(*) 15. (a) Show that if $B \subseteq \mathbb{R}$ is *bounded*, and $A \subseteq B$, then A is bounded.

(*) (b) If $S \subseteq \mathbb{R}$, then we define the set $|S|$ as $|S| = \{|s| \ : \ s \in S\}$. Show that if S is bounded, then $|S|$ is bounded.

16. For each of the following sets, either show that it is bounded (and find bounds), or explain why it isn't. [You can appeal to results from calculus in your answers.]

(a) $A = \left\{ \sum_{k=1}^{n} \frac{1}{k} \ : \ n \in \mathbb{N} \right\}$

(b) $B = \left\{ \sum_{k=1}^{n} \frac{1}{2^k} \ : \ n \in \mathbb{N} \right\}$

(c) $C = \left\{ \frac{\ln n}{n} \ : \ n \in \mathbb{N} \right\}$

(d) $D = \left\{ \frac{2^n}{n^2} \ : \ n \in \mathbb{N} \right\}$