Math 325 Problem Set 8

Starred (*) problems are due Friday, October 26.

45. Show that the function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x \cdot |x|$ is differentiable at every $a \in \mathbb{R}$.

    [Hint: in most cases, you can (if you stay close to a) replace f with a more 'amenable' function...]

46. (Belding and Mitchell, p.99, #3(b)) Show that if $0 \in D$ and $f : D \to \mathbb{R}$ is continuous at $a = 0$, then $g : D \to \mathbb{R}$ given by $g(x) = xf(x)$ is differentiable at $a = 0$.

(*) 47. (Belding and Mitchell, p.100, #10 (sort of)) If $a \in D$ and $f : D \to \mathbb{R}$ is differentiable at a <u>and</u> $f'(a) > 0$, show that there is a $\delta > 0$ $x \in (a, a + \delta)$ implies that $f(x) > f(a)$ and $x \in (a - \delta)$ implies that $f(x) < f(a)$.

(*) 48. Show that if $a \in D$, $f, g : D \to \mathbb{R}$ are both differentiable and $x = a$, $f(a) = g(a)$, <u>and</u> $f(x) \leq g(x)$ for all $x \in D$, then $f'(a) = g'(a)$.

    [What's the alternative? The previous problem can help!]

49. (The 'Squeeze Play Theorem' for derivatives): Show that if $a \in D$, $f, g, h : D \to \mathbb{R}$ are functions with $f(x) \leq g(x) \leq h(x)$ for all $x \in D$, $f(a) = g(a) = h(a)$, <u>and</u> f and h are differentiable at $x = a$, then g is differentiable at $x = a$ and $f'(a) = g'(a) = h'(a)$.

    [Note that the only 'new' things here are that g is differentiable at a and $g'(a) = f'(a)$ ...]

(*) 50. Suppose that $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are both continuous, and f is differentiable at $x = 0$, with $f(0) = f'(0) = 0$. Show that $h(x) = f(x)g(x)$ is also differentiable at $x = 0$ and $h'(0) = 0$.

    [Note that since we do <u>not</u> know that g is differentiable at $x = 0$, we <u>cannot</u> use the product rule (even if we knew what that was)....]

51. As almost none of us learn, the angle sum formula for tangent is

$$\tan(a + h) = \frac{\tan a + \tan h}{1 - \tan a \tan h}$$

    Use this to show directly from the ("limit as $h \to 0$" definition) that the derivative of $f(x) = \tan x$ is what you were told it is in calculus class.

    [If you want something extra to do, derive this angle sum formula from the angle sum formulas for $\sin x$ and $\cos x$ (for fun!).]