

## Math 325 Elementary Analysis

### List of topics covered

Final Exam Monday April 30, 3:30pm-5:30pm, Oldfather 208

#### The Real Numbers

the real numbers; the axioms of a field  
natural numbers, integers, rationals, reals  
ordering; triangle inequality  
least upper bounds, greatest lower bounds; completeness  
the Archimedean property; mathematical induction

#### Sequences

sequences, convergence ( $\epsilon$ - $N$ )  
arithmetic operations on sequences  $(+, -, \cdot, /)$ ,  $\leq$   
monotone sequences; bounded implies convergent!  
Cauchy sequences; Cauchy implies convergent!  
subsequences; every sequence has a monotone subsequence!  
limsup, liminf; convergent  $\Leftrightarrow$  liminf = limsup

#### Limits of functions

definition of limit,  $\epsilon$ - $\delta$  proofs  
accumulations points; uniqueness of limits  
finding limits using sequences; nonexistence of limits using sequences!  
arithmetic operations and limits  $(+, -, \cdot, /)$ ,  $\leq$

#### Continuity

continuity at a point, relation to sequences  
continuity and  $(+, -, \cdot, /)$ , composition  
Intermediate value theorem; root finding  
Extreme value theorem; max/min  
uniform continuity; unif cts implies cts!

#### Integration

partitions, upper and lower Riemann sums  $U(f, P)$ ,  $L(f, P)$   
Riemann integrability; partition with small mesh implies  $U(f, P) - L(f, P)$  small  
continuous implies integrable; monotone implies integrable  
Fundamental Theorem of Calculus

#### Uniform Convergence

derivatives; mean value theorem  
intermediate value theorem for derivatives  
failures of continuity: holes, jumps, no one-sided limit  
pointwise limit of a sequence of functions; Baire class one functions  
uniform convergence  
continuity, uniform continuity, integrability are preserved by uniform convergence  
Weierstrass  $M$ -test  
    application: power series  
radius of convergence  
term-by-term integration  
term-by-term differentiation  
continuous functions that are differentiable nowhere!