Math 325 Practice Exam 1 Problems

Show all work. (I.e., if you think it, write it!)

1. Use induction to show that for every n≥1,

$$a_n = \sum_{k=1}^{n} \frac{1}{k(k+2)} = \frac{n(3n+5)}{4(n+1)(n+2)} = \text{f}(n).$$

(Hint: write out what f(n+1) is; it'll help.

2. Use the Rational Roots Theorem to show that $r = \sqrt{2} - \sqrt{5}$ is a not a rational number.

3. Find the limit of the sequence $a_n = \dfrac{n^2 - n + 1}{3n^2 - 1}$

and prove you are right using the ϵ-N definition of the limit. [Also: show how to do this quicker using our limit theorems!]

4. Suppose that S and T are both non-empty subsets of the real line, and both are bounded from above. Show that <u>either</u> $\sup(S \cup T) \leq \sup(S)$ <u>or</u> $\sup(S \cup T) \leq \sup(T)$.

(Hint: <u>Either</u> $\sup(S) \leq \sup(T)$ <u>or</u> $\sup(T) \leq \sup(S)$. In each case, show that one of the two statements above must be true. (25 pts.))

[Note: $S \cup T = \{x \in \mathbb{R} : x \in S \text{ or } x \in T\}$.]

1. Define a sequence as follows: let a_1=1, and for $n > 1$ let $a_n = a_{n-1} + \dfrac{1}{n^2}$

Show that a_n is a monotone increasing sequence that is bounded from above. What doe this tell us about the sequence?

4. Let a_1=2, and for n≥1 define a_n (inductively) by $a_{n+1} = \frac{1}{2}(a_n + \frac{1}{a_n})$.

(a) **ASSUME** that $a_n > 1$ for all n, and show that $\{a_n\}_{n=1}^{\infty}$ is a monotone decreasing sequence. (Hint: write out $a_n - a_{n+1}$ and show that it is positive!)

(b) Conclude that the sequence converges, and determine its limit. (Hint: $\lim_{n\to\infty} a_n = \lim_{n\to\infty} a_{n+1}$.)

5. Show, using the **definition** of convergence, that $\displaystyle \lim_{n\to\infty} \frac{n^2+1}{2n^2+n} = \frac{1}{2}$

3. Let a_1=1, and define a_n, inductively, by $a_{n+1} = \frac{2}{3}(a_n + 5)$

(a): Show by induction that $a_n \leq 10$ for all n.

(b): Use (a) to show that $a_{n+1} \geq a_n$ for all n.

(c): Conclude that the sequence $(a_n)_{n=1}^{\infty}$ converges! (5 pts.)

1. Show that a=$\sqrt{\sqrt{2}+1}$ is not a rational number.