Math 325 Problem Set 1

Due Monday, Jan. 23

1. Working from our axioms for the ordered field $\mathbb{R}$, show that if $x, y \in \mathbb{R}$ and $x < y$, then $x < \dfrac{x+y}{2} < y$.

2. [Lay, p. 115, # 11.3 (c,d,f)] Show:

   (α) If $x \neq 0$, then $\dfrac{1}{x} \neq 0$ and $\dfrac{1}{(1/x)} = x$.

   (β) If $xy = xz$ and $z \neq 0$, then $x = y$.

   (γ) $0 < 1$.

3. Working from our axioms for the ordered field $\mathbb{R}$, show that for any $x \in \mathbb{R}$, $x^2 + 1 > 0$.

   [N.B.: In particular, $x^2 + 1 = 0$ has no solution. This shows that the complex numbers $\mathbb{C}$ cannot support an order '$<$' making $\mathbb{C}$ an ordered field.]