Math 325 Problem Set 4

Due Friday, February 17

12. [Lay, p.164, # 16.7(f) (sort of)] Show that if $0 \leq x < 1$ then for any $\epsilon > 0$ there is an $n \in \mathbb{N}$ so that $x^n < \epsilon$. [Hint: Suppose not! Then look at lower bounds for $A = \{x^n : n \in \mathbb{N}\}$.] Conclude that for <u>every</u> $m \in \mathbb{N}$ with $m \geq n$ we have $|x^m| = x^m < \epsilon$, so $x^n \to 0$ as $n \to \infty$.

13. [Lay, p.165, # 16.13] (The 'Squeeze Play' Theorem) Suppose that $(a_n)_{n=1}^{\infty}$, $(b_n)_{n=1}^{\infty}$, and $(c_n)_{n=1}^{\infty}$ are sequences with $a_n \leq b_n \leq c_n$ for all n. Suppose further that $\lim\limits_{n\to\infty} a_n = L = \lim\limits_{n\to\infty} c_n$. Show that $\lim\limits_{n\to\infty} b_n = L$.

14. Show, from the definition of limit (i.e., no limit theorems!) that

    (a) $\displaystyle\lim_{n\to\infty} \frac{2n+1}{3n+2} = \frac{2}{3}$ and (b) $\displaystyle\lim_{n\to\infty} \frac{n^2+n-2}{2n^2+n-1} = \frac{1}{2}$

15. Show that if $a_n \geq 0$ for all $n \in \mathbb{N}$ and $\lim\limits_{n\to\infty} (-1)^n a_n = L$, <u>then</u> $L = 0$.
    [Hint: Suppose not!]