Math 325 Problem Set 1

Problems are due Friday, January 20.

1. [Zorn, p.13 #2] Let $S = \{x \in \mathbb{R} \mid x^2 + x = 0\}$ and $T = \{x \in \mathbb{R} \mid x^2 + x < 5\}$.

   (a) Write S and T as (small) unions of points and/or intervals.

   (b) Decide whether each of the following statements is true, and (briefly) explain:
   $$S \subseteq \mathbb{N} \; ; \; S \subseteq T \; ; \; T \cap \mathbb{Q} \neq \emptyset \; ; \; -2.8 \in \mathbb{Q} \setminus T \; .$$

   (c) Describe the set $U = \{s \in \mathbb{R} \mid x^2 + x < 0\}$ as a union of intervals.

2. [Zorn, p.14, #10] Starting with a set S, we can construct a new set $P(S)$, the <u>power</u> <u>set</u> of S, consisting of all subsets of S. For example, $P(\{1,2\}) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$.

   (a) Find $P(\{1,2,3\})$.

   (b) Show that if $S \subseteq T$, then $P(S) \subseteq P(T)$.

   (c) If we set $N_k = \{1, 2, \ldots, k\}$, explain why $P(N_{11})$ has twice as many elements as $P(N_{10})$.

3. [Zorn, p.26, #8] Let L be the (linear) function $L(x) = ax + b$, where a and b are (real) constants and $a \neq 0$.

   (a) Explain why L is both one-to-one and onto.

   (b) Find a formula for the inverse function $M = L^{-1}$, and show that
   $L \circ M(x) = M \circ L(x) = x$ for every $x \in \mathbb{R}$.

4. [Zorn, p.26, #10 (part)] Suppose the $f : A \to B$ and $g : B \to C$ are both functions.

   (a) Show that if f and g are both one-to-one, then $g \circ f : A \to C$ is one-to-one.

   (b) Show that if $g \circ f$ is onto, then g is onto.