Math 325 Problem Set 10 (and last...)

Problems are due Friday, April 21.

36. ['another' L'Hôpital's Rule] Show that if $f(x) \to 0$ and $g(x) \to 0$ as $x \to \infty$, and

$$\frac{f'(x)}{g'(x)} \to L \text{ as } x \to \infty, \text{ then } \frac{f(x)}{g(x)} \to L \text{ as } x \to \infty \ .$$

[Hint: Look at Proposition 3.6(ii), as a way to convert this into an 'ordinary' L'Hôpital's Rule problem...]

37. [Zorn, p.226, # 9] Show that if f is integrable on $[a, b]$, and you can show (from the definition!) that $\int_a^b f(x)\ dx = L$ and $\int_a^b f(x)\ dx = M$, then $L = M$. [I.e., 'the value of an integral is unique'.]

[Suppose not! Show that there is a partition P that gets you into trouble...]

38. [Zorn, p.236, # 1]

(a): Show that if h is integrable on the interval $[a, b]$ and $h(x) \geq 0$ for every $x \in [a, b]$, then for every partition $P = \{a = x_0 < x_1 < \cdots < x_n = b\}$ of $[a, b]$ and set of 'samples' $S = \{c_1, \ldots, c_n\}$ with $x_{i-1} \leq c_i \leq x_i$ for each i, we have $R(h, P, S) \geq 0$. Explain why we can then conclude that $\int_a^b h(x)\ dx \ \geq \ 0$.

(b): Use part (a) and the properties of integrals (Theorem 5.5) to show that if f and g are integrable on $[a, b]$ and $f(x) \geq g(x)$ for every $x \in [a, b]$, then $\int_a^b f(x)\ dx \ \geq \int_a^b g(x)\ dx$.

39. [Zorn, p.236, # 2] Suppose that f is integrable on $[a, b]$, and $m \leq f(x) \leq M$ for every $x \in [a, b]$. Show that $m(b - a) \leq \int_a^b f(x)\ dx \leq M(b - a)$.