Math 325 Problem Set 2

Problems are due Friday, January 27.

5. [Zorn, p.36, #3] For the statements (a)-(d) below, state both the converse and the contrapositive of the given statement, and indicate (no explanation needed) which of statement, converse, and contrapositive are <u>true</u>.

   (a) If $a \in \mathbb{Q}$ and $b \in \mathbb{Q}$, then $a + b \in \mathbb{Q}$.

   (b) If $a \notin \mathbb{Q}$, then $\dfrac{1}{a} \notin \mathbb{Q}$.

   (c) If $a \notin \mathbb{Q}$ and $b \notin \mathbb{Q}$, then $ab \notin \mathbb{Q}$.

   (d) If $a_n \in \mathbb{R}$ for all $n \in \mathbb{N}$ and $\displaystyle\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.

6. Show, using the Rational Roots Theorem, that $\alpha = \sqrt{2 + \sqrt{3}}$ is not rational.

   [Find a polynomial with integer coefficients that has α as a root!]

7. [Zorn, p.45, #6] By looking at the first few cases, find a (short) formula for the sum

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{n(n+1)} = \sum_{k=1}^{n} \frac{1}{k(k+1)} \ ;$$

   then prove, using induction, that your formula is correct.

8. Show, by induction, that if $x > 0$ then $(1 + x)^n \geq nx + 1$ for every $n \in \mathbb{N}$.