Math 325 Problem Set 3

Problems are due Friday, February 3.

9. [Zorn, p.58, #4] Show, by induction, that the (ordinary) triangle inequality extends to show that for any $n \geq 2$ we have

$$\left| \sum_{k=1}^{n} a_k \right| \leq \sum_{k=1}^{n} |a_k| \ .$$

10. [Zorn, p.58, #6] Show that the maximum of two numbers $x, y \in \mathbb{R}$ can be computed by

$$\max(x, y) = \frac{x + y + |x - y|}{2} \ .$$

[That is, if $x \leq y$ then $\frac{x + y + |x - y|}{2} = y$, while if $y \leq x$ then it equals x.]

Find a similar formula which gives the minimum of x and y.

11. [Zorn, p.64, #7] (a) Show that if $B \subseteq \mathbb{R}$ is bounded, and $A \subseteq B$, then A is bounded.

(b) If $S \subseteq \mathbb{R}$, then we define the set $|S|$ as $|S| = \{|s| \ : \ s \in S\}$. Show that if S is bounded, then $|S|$ is bounded.

12. [Zorn, p.64, #8] For each of the following sets, either show that it is bounded (and find bounds), or explain why it isn't. [You can appeal to results from calculus in your answers.]

(a) $A = \left\{ \sum_{k=1}^{n} \frac{1}{k} \ : \ n \in \mathbb{N} \right\}$

(b) $B = \left\{ \sum_{k=1}^{n} \frac{1}{2^k} \ : \ n \in \mathbb{N} \right\}$

(c) $C = \left\{ \frac{\ln n}{n} \ : \ n \in \mathbb{N} \right\}$

(d) $D = \left\{ \frac{2^n}{n^2} \ : \ n \in \mathbb{N} \right\}$