Math 325 Problem Set 4

Problems are due Friday, February 10.

13. [Zorn, p.64, #11] Our text says that a set A is *bounded away from* 0 if there is an $\epsilon > 0$ so that for every $x \in A$ we have $|x| > \epsilon$. Show that A is bounded away from 0 if and only if the set $B = \{\frac{1}{x} \mid x \in A\}$ is bounded.

    [N.B. "P if and only if Q" means P implies Q <u>and</u> Q implies P.]

14. If we set $A = \{x \in \mathbb{R} \mid x^3 < 2\}$, show that A is bounded above, so has a supremum $\alpha = \sup(A)$. Then show (in a manner similar to our classroom demonstrations) that $\alpha^3 < 2$ is not possible. (If you are feeling like doing even more, show that $\alpha^3 > 2$ is also impossible! From that, we can conclude that $\alpha^3 = 2$.)

15. For subsets $A, B \subseteq \mathbb{R}$, we define their 'sum' $A + B = \{a + b \ : \ a \in A, b \in B\}$.

    Show that if A and B are both non-empty and bounded from above, then so is $A + B$, and
    $$\sup(A + B) = \sup(A) + \sup(B) \ .$$

    [Hint: show that $\sup(A) + \sup(B)$ is an upper bound! Then worry about whether there might be a smaller one...]