Math 325 Problem Set 5

Problems are due Friday, February 17.

16. [Zorn, p.81, #8 (part)] For the sequences below, determine the limit of the sequence and prove your assertion using the ϵ - N formulation of the limit. [You may use the fact that $|\sin x| \leq 1$ for all $x \in \mathbb{R}$.]

    (a) $a_n = \dfrac{2n}{3n-5}$ (b) $b_n = \dfrac{2n}{3n+\sin n+5}$

17. Show that if $a_n \to L$ as $n \to \infty$, then $|a_n| \to |L|$ as $n \to \infty$. Show, in fact, that for a notion of small, $\epsilon > 0$, the same $N \in \mathbb{N}$ that works to control a_n will work to control $|a_n|$.

    [The results of section 1.7 will help with this.]

18. [Zorn, p.89, #2] Show that if $b_n \neq 0$ for every $n \in \mathbb{N}$, $\lim\limits_{n\to\infty} b_n = b$, and $b \neq 0$, then
$$\lim_{n\to\infty} \frac{1}{b_n} = \frac{1}{b} \ .$$

    [The textbook describes one possible approach; one could also do this without breaking it into so many steps....]

19. Explain how our results about how convergent sequences combine (from section 2.2) enable us to determine the limit of the sequence
$$\frac{2n^2 + n\sin n - 9}{3n - 5n^2 + 2^{-n}} \ .$$