Math 325 Problem Set 6

Problems are due Friday, March 3.

20. [Zorn, p.99, #9] Suppose $(a_n)_{n=1}^{\infty}$ is a sequence and $L \in \mathbb{R}$. Show that if $a_{n_k} \to L$ for every <u>monotonic</u> subsequence of $(a_n)_{n=1}^{\infty}$, then $a_n \to L$.

21. Show *directly* (i.e., without quoting "Cauchy implies convergent" and "convergent implies Cauchy") that if a_n and b_n are Cauchy sequences, then so are the sequences $c_n = a_n + b_n$ and $d_n = a_n b_n$. [Hint: for the second, you will need to use Cauchy implies bounded?]

22. A sequence a_n is called *contractive* if for some constant $0 < k < 1$ we have $|s_{n+2} - s_{n+1}| < k|s_{n+1} - s_n|$ for all $n \in \mathbb{N}$. Show that every contractive sequence is Cauchy (and therefore converges).

    [Hint: By induction, $|s_{n+2} - s_{n+1}| < k^n |s_2 - s_1|$, and $\displaystyle\sum_{r=m+1}^{n} k^r$ is something (from calculus!) we know the exact value of...]

23. [Zorn, p.144, #2 (sort of)] Use calculus to 'determine' the following limits, then use the $\epsilon - \delta$ definition of limit to prove that you are correct:

    (α) $\displaystyle\lim_{x \to 1} 2x + 3$

    (β) $\displaystyle\lim_{x \to 2} \frac{1}{2x + 3}$