Math 325 Problem Set 7

Problems are due Friday, March 10.

24. Show that for every $a \in \mathbb{R}$ we have $\lim_{x \to a} x = a$. (Your argument should be quite short!) Then, using induction and our limit theorems, show that for every $n \in \mathbb{N}$ we have $\lim_{x \to a} x^n = a^n$.

25. (a) Show that the (not very well-known? but it follows from angle sum formulas) trig identity

$$\sin(A) - \sin(B) = 2 \sin\left(\frac{A-B}{2}\right) \cos\left(\frac{A+B}{2}\right) \qquad \text{for every } A, B \in \mathbb{R},$$

and the fact (which we will show in class!) that $|\sin(C)| \leq |C|$ for every $C \in \mathbb{R}$, together imply that for every $x, y \in \mathbb{R}$ we have $|\sin(x) - \sin(y)| \leq |x - y|$.

(b) Using (a), show that the function $f(x) = \sin x$ is continuous at a for every $a \in \mathbb{R}$.

26. Show that if $f : (a, b) \to \mathbb{R}$ is a continuous function, then the function $g : (a, b) \to \mathbb{R}$ given by $g(x) = |f(x)|$ is also continuous. (You should argue directly from ϵ's and δ's.)

27. Using the previous problem (and a problem from a previous problem set!), show that if $f, g : (a, b) \to \mathbb{R}$ are continuous functions, then the function $M : (a, b) \to \mathbb{R}$ given by $M(x) = \max\{f(x), g(x)\}$ is also continuous.