Math 325 Problem Set 8

Problems are due Friday, March 17.

28. [Zorn, p.152, # 1] Suppose that $f : \mathbb{R} \to \mathbb{R}$ is continuous and $f(x) = 0$ for every $x \in \mathbb{Q}$. Show that $f(x) = 0$ for every $x \in \mathbb{R}$.

29. Using the problem above, show that if $f, g : \mathbb{R} \to \mathbb{R}$ are both continuous functions, and $f(x) = g(x)$ for every $x \in \mathbb{Q}$, then $f = g$ (i.e., $f(x) = g(x)$ for every $x \in \mathbb{R}$).

    [‘A continuous function is <u>determined</u> by its values on the rational numbers.’]

30. [‘Pasting’ continuous functions together.] Show that if $a < b < c$ and if $f : [a, b] \to \mathbb{R}$ and $g : [b, c] \to \mathbb{R}$ are both continuous functions, and $f(b) = g(b)$, then the function $h : [a, c] \to \mathbb{R}$ defined by

$$h(x) = \begin{cases} f(x) & \text{if } x \leq b \\ g(x) & \text{if } x \geq b \end{cases}$$

    is continuous at $x = b$. Why is it also continouous at every <u>other</u> point in $[a, c]$?

31. [Zorn, p.154, #10] Suppose that $a < 0 < b$ and $f : (a, b) \to \mathbb{R}$ is a function that is bounded (i.e., for some $M \in \mathbb{R}$, $|f(x)| \leq M$ for every $x \in (a, b)$). Show that the function $g : (a, b) \to \mathbb{R}$ defined by $g(x) = xf(x)$ is continuous at $x = 0$. Show, on the other hand, that for any <u>other</u> $c \in (a, b)$ we have that g is continuous at c <u>if</u> <u>and</u> <u>only</u> <u>if</u> f is continuous at c.

    [The last assertion can be attacked using ‘general’ results we have established, <u>or</u> directly using ϵ’s and δ’s (your choice!).]