Math 325 Problem Set 9

Problems are due Friday, April 14.

32. [Zorn, p.182, # 6] Suppose that $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are both continuous, and f is differentiable at $x = 0$, with $f(0) = f'(0) = 0$. Show that $h(x) = f(x)g(x)$ is also differentiable at $x = 0$ and $h'(0) = 0$.

    [Note that since we do not know that g is differentiable at $x = 0$, we cannot use the product rule....]

33. As almost none of us learn, the angle sum formula for tangent is

$$\tan(a+h) = \frac{\tan a + \tan h}{1 - \tan a \tan h}$$

    Use this to show directly from the ("limit as $h \to 0$" definition) that the derivative of $f(x) = \tan x$ is what you were told it is in calculus class.

    [If you want something extra to do, derive this angle sum formula from the angle sum formulas for $\sin x$ and $\cos x$ (for fun!).]

34. [Zorn, p.193, # 2 (parts)]

    (a) Use the product and chain rules to derive a general formula for the second derivative $(f \circ g)''(x)$; you should assume that $f''(x)$ and $g''(x)$ both exist.

    (b) Find a 'hybrid product-chain rule' to express the derivative $(f \circ (gh))'(x)$; you should assume that f, g and h are all differentiable.

35. [Zorn, p.200, # 4] Use Rolle's Theorem to show, by induction, that a polynomial $p(x) = a_n x^n + \cdots + a_1 x + a_0$ of degree n has at most n distinct roots (i.e., solutions to $p(x) = 0$).

    [Hint: If p has degree n, then p' has degree $n - 1$ ...]