Math 417 Problem Set 2

Starred (*) problems are due Friday, September 7.

7. (Gallian, p.56, #31) Show that for any group G, its 'Cayley' table is a *Latin square*: every group element appears exactly once in each row and column of the table.

8. (Gallian, p.24, #19) Show that $\gcd(n, ab) = 1$

$$\text{if and only if } \gcd(n, a) = 1 \text{ and } \gcd(n, b) = 1.$$

[This is what 'makes' $\mathbb{Z}_n^*$ a group under multiplication; the product of two numbers relatively prime to n is a number relatively prime to n.]

(*) 9. Use the Euclidean algorithm to find the inverses of the elements 2, 5, and 7 in the group $G = (\mathbb{Z}_{141}^*, \cdot, 1)$.

(*) 10. Find the inverse of the element $\begin{pmatrix} 2 & 6 \\ 3 & 5 \end{pmatrix}$ in $GL_2(\mathbb{Z}_{11})$.

11. (Gallian, p.57, #42) Suppose that $F_1 = M(\theta)$ and $F_2 = M(\psi)$ (in Gallian's/our notation) are reflections in lines through the origin of slope θ and ψ, with $\theta \neq \psi$, and $F_1 \circ F_2 = F_2 \circ F_1$. Show that then $F_1 \circ F_2 = R(\pi)$ is rotation by angle π.

[Your results from Problem #1 might help!]

(*) 12. (Gallian, p.57, #34) Prove that if G is a group and $a, b \in G$ then $(ab)^2 = a^2b^2$ if and only if $ab = ba$.

13. (Gallian, p.58, #47) Suppose that G is a group and, for every $x \in G$, we have $x^2 = e$. Show that for every $a, b \in G$ we have $ab = ba$ (that is, G is abelian!).