Math 417 Problem Set 3

Starred (*) problems are due Friday, September 14.

14. Give an example of a group G and $a, b \in G$ so that $(ab)^4 = a^4 b^4$, but $ab \neq ba$.

    [Hint: Problem #11 might help? Slightly bigger challenge: try the same thing with the 4's replaced by 3's !]

(*) 15. (Gallian, p.58, #51) Show that, if we had 'weakened' the definition of a group G to (1) there is an $e \in G$ with $ge = g$ for every $g \in G$, (2) inverses exist, and (3) the group operation is associative, then we can prove that $eg = g$ for every $g \in G$ (i.e, the other half of the definition of an identity automatically holds).

16. (Gallian, p.57, #43, sort of) Show that if R is a rotation of $\mathbb{R}^2$ around the origin, and M is a reflection over a line through the origin, then for any $k \in \mathbb{Z}$ we have $R^k M R^k = M$.

    [Hint: Problem #1 might help!]

(*) 17. (Gallian, p.57, #39) If G is a group, and for every $a, b, c, d, x \in G$ we have $axb = cxd$ implies that $ab = cd$, show that then for every $u, v \in G$ we have $uv = vu$. ('A middle cancellation law implies commutativity.')

    [Hint: Find an x so that $uxv = vxu$!]

18. (Gallian, p.69, # 4) Show that if G is a group and $a \in G$, then $|a| = |a^{-1}|$.

(*) 19. If G is a group and $a \in G$, and if $|a| < \infty$ and $\gcd(k, |a|) = 1$, show that then $|a^k| = |a|$.

20. (Gallian, p.74, #77) Show that if G is a group with $g \in G$ and $n = |g| < \infty$, and k is relatively prime to n, then there is an $h \in G$ with $g = h^k$.

    [Hint: This should be true even if we replace G with a subgroup of G which contains g, e.g., $H = \langle g \rangle$!]