Name:

Math 423/823 Exam 1

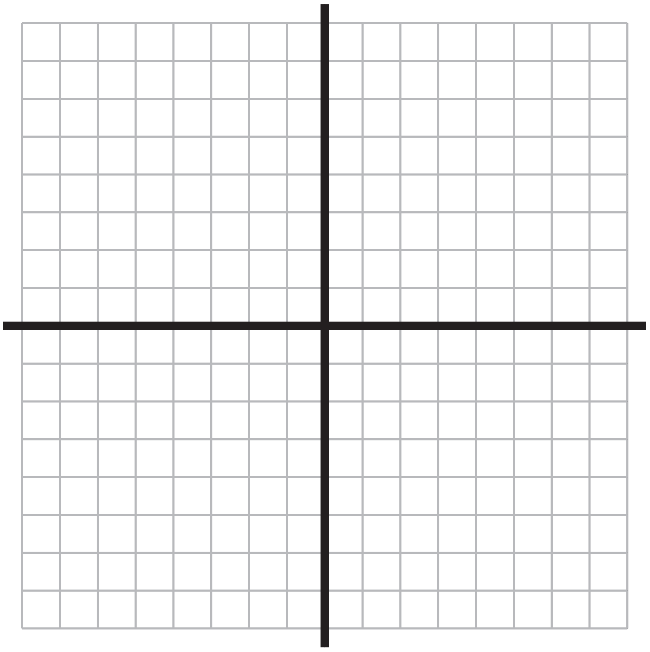
Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

1. (15 pts.) Solve for $z = x + yi$:

$$(1+i)(2+i) = (5+2i)z$$

2. (15 pts.) Sketch the collection of points which satisfy the equation

$$\mathrm{Re}(z \cdot \overline{z} + 4zi) = 5$$



3. (20 pts.) Find the 3-rd roots of $z = 2i$ (i.e, the w for which $w^3 = z$).

[You can write them in either $w = x + yi$ or $w = re^{i\theta}$ form.]

4. (10 pts. each) Find each of the indicated limits (or show that it doesn't exist):

(a): $\lim\limits_{z\to i} \dfrac{z^3 - z + 2i}{z^2 + 1}$

(b): $\lim\limits_{z\to\infty} e^{-z^2}$

5. (15 pts.) Show that the function $f(z) = (\overline{z})^3$ is analytic at <u>no</u> value of z.

[Hint: You can argue directly, or use the fact that $g(z) = z^3$ <u>is</u> analytic.]

6. (15 pts.) Find an entire function $f(x, y) = u(x, y) + iv(x, y)$ for which

$$u(x, y) = x^2 + 2xy - y^2$$

[That is, find a harmonic conjugate of $u(x, y)$.]