Math 423/823 Exam 2

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

You may use your notes and the course text as reference while completing this exam. You may consult no other resource while working on the exam, with the exception of your instructor. This is *not* as a test of reading comprehension; we want you to be certain that you are answering the intended question. If something seems unclear, consult your instructor! (Email will likely be fastest.)

Note that the exam consists of five (5) problems. **Your completed exam is due on the desk in front of your instructor at the beginning of class (9:32am) on Thursday, April 14.**

1. (20 pts.) Find all values of $z \in \mathbb{C}$ for which $\sin(z) = i$.

2. (20 pts.) Find the value of $\displaystyle\int_C f(z)\,dz$, where $f(z) = f(x+iy) = x^2 - iy^2$ and $C(t) = e^{it}$ for $0 \leq t \leq \pi$.

3. (20 pts.) Find the integral of the function $f(z) = \dfrac{z}{z^3+1}$ around the simple closed curve $C(t) = [3+\sin(5t)]\cos t + i[3+\sin(2t)]\sin t$, $0 \leq t \leq 2\pi$. [See figure below.]

4. (20 pts.) Let C be a simple closed curve oriented counterclockwise, f a function which is analytic on and inside of the curve C, and z_1, z_2 two (distinct!) points lying inside of C. Show that
$$\int_C \frac{f(z)}{(z-z_1)(z-z_2)}\,dz = \frac{f(z_1)-f(z_2)}{z_1-z_2}$$

5. (20 pts.) If $w = f(z)$ is analytic **and non-constant** on and inside of the simple closed curve C and, for some constant K, $|f(z)| = K$ for every point on C, show that there is a point z_0 inside of C where $f(z_0) = 0$.

   [Hint: Suppose not! Then show that we can apply the Maximum Principle to both $f(z)$ and $g(z) = \dfrac{1}{f(z)}$ and get ourselves into trouble!]