Math 423/823 Homework 1

Due Thursday, Jan. 27

1. [BC#1.2.6(b)] For complex numbers $z_1 = a_1 + b_1 i$, etc., verify the distributive law:

$$z_1(z_2 + z_3) = z_1 z_2 + z_1 z_3$$

[N.B.: probably better to write things as $z_1 = (a_1, b_1)$, etc. and expand out both sides, to avoid inadvertently 'assuming' something?]

2. [BC#1.3.1] Reduce each of the quantities to a real number:

(a) $\dfrac{1 + 2i}{3 - 4i} + \dfrac{2 - i}{5i}$ (c) $(1 - i)^4$

3. [BC#1.5.11] Use mathematical induction to show that for all natural numbers n, and complex numbers $z_1, \ldots, z_n$,

$$\overline{z_1 + \cdots + z_n} = \overline{z_1} + \cdots + \overline{z_n} \qquad \text{and} \qquad \overline{z_1 \cdots z_n} = \overline{z_1} \cdots \overline{z_n}$$

4. Show that if $p(x) = a_n x^n + \cdots + a_0$ is a polynomial with real coefficients, and $z = a + bi$ is a complex root of p [i.e., $p(z) = a_n z^n + \cdots + a_0 = 0$], then $\overline{z}$ is also a root of p.