Math 423/823 Exercise Set 2

Due Thursday, Feb. 3

5. [BC#1.8.1 sort of] Find the exponential form ($re^{i\theta}$) of the following numbers:

   (a) $z = \dfrac{i}{-2 - 2i}$

   (b) $z = (\sqrt{3} - i)^6$

6. [BC#1.8.8] Show that for complex numbers z_1 and z_2 we have $|z_1| = |z_2|$ if and only if there are complex numbers c_1 and c_2 so that $z_1 = c_1 c_2$ and $z_2 = c_1 \overline{c_2}$.

   [N.B.: for $\Rightarrow$, show that we if $|z_1| = |z_2|$ then can write $z_1/z_2 = e^{i\theta}$ and use this to construct an appropriate $c_2 = e^{i\psi}$. Or you can unravel the hints provided in the text...]

7. [BC#1.10.3] Find all of the roots indicated:

   (a) $(-1)^{1/3}$

   (b) $8^{1/6}$

8. Show that $(z - z_0)(z - \overline{z_0}) = z^2 - 2\text{Re}(z_0)z + |z_0|^2$, which has real coefficients. Use this and the results of Problem #7 to express the polynomial $p(x) = x^6 - 8$ as a product of linear and quadratic polynomials with real coefficients.