

### Math 423/823 Exercise Set 3

Due Thursday, Feb. 10

9. [BC#2.12.3] If  $z = x + yi$  and  $f(z) = (x^2 - y^2 - 2y) + (2x - 2xy)i$ , use the formulas

$$x = \frac{z + \bar{z}}{2} \quad \text{and} \quad y = \frac{z - \bar{z}}{2i}$$

to write  $f(z)$  in terms of  $z$  (and  $\bar{z}$ ) and simplify the result.

10. [BC#2.14.3] Sketch the regions onto which the sector

$$A = \{z = re^{i\theta} : 0 \leq r \leq 1, 0 \leq \theta \leq \pi/4\}$$

is mapped by the functions

(a)  $w = z^2$       (b)  $w = z^3$       (c)  $w = z^4$

11. Show that the reciprocal function,  $f(z) = 1/z$ , maps the (punctured) disk

$$D = \{z : |z - 1| < 2 \text{ and } z \neq 0\}$$

onto the region that lies outside of the circle  $\{w : |w + 1/3| = 2/3\}$ .

[N.B. Essentially, this is asking you to show that  $|z - 1| < 2 \Leftrightarrow |1/z + 1/3| > 2/3$ .

The quickest way that I found to do this was to start with  $z = x + yi$  and  $|1/z + 1/3|^2 > (2/3)^2$ , clear the denominator and continue to simplify the expression until I was staring at  $|z - 1|^2 < 4$  ...]

12. Find  $\lim_{z \rightarrow 1+i} \frac{z^2 + z - 1 - 3i}{z^2 - 2z + 2}$ .