

Math 423/823 Exercise Set 5

Due Thursday, Mar. 10

17. [BC#2.26.7] Let the function $f(z) = u(x, y) + iv(x, y)$ be analytic in a domain D and consider the families of level curves

$$\mathcal{U} = \{(x, y) : u(x, y) = c_1\} : c_1 \in \mathbb{C} \text{ and } \mathcal{V} = \{(x, y) : v(x, y) = c_2\} : c_2 \in \mathbb{C}.$$

Show that wherever they meet, the curves in \mathcal{U} are orthogonal to the curves in \mathcal{V} . That is, the slopes of the two curves, at a point of intersection, are negative reciprocals.

[Hint: for each curve treat it as implicitly defining y as a function of x and use the multivariate chain rule to, e.g., differentiate both sides of $u(x, y(x)) = c_1$ w.r.t. x .]

18. [BC#3.29.12] For $z = x + yi$, write $\operatorname{Re}(e^{1/z})$ in terms of x and y . Explain why this function is harmonic in every domain D that does not contain 0.

19. [BC#3.31.5] Show that:

(a): the set of values of $\log(i^{1/2})$ is $(n + \frac{1}{4})\pi i$ for n any integer, and that the same is true for $\frac{1}{2} \log(i)$

(b): the set of values of $\log(i^2)$ is not the same as the set of values for $2 \log(i)$.

20. For $z = x + yi$, does 1^z always equal 1 ?