Math 423/823 Exercise Set 6

Due Thursday, Mar. 17

21. [BC#4.38.4] The integral $\int_0^\pi e^{(1+i)x}\ dx$ is, technically, equal to

$$\int_0^\pi e^x \cos x\ dx + i \int_0^\pi e^x \sin x\ dx$$

Evaluate these two integrals $\int_0^\pi e^x \cos x\ dx$ and $\int_0^\pi e^x \sin x\ dx$ by applying the Fundamental Theorem of Calculus (p.119, bottom) directly to the top integral and equating the real and imaginary parts.

22. Find a parametriation of the curve which follows the circle of radius 2 counterclockwise from $z = 2$ to $z = 2i$, followed by the line segment that runs from $z = 2i$ to $z = -1$.

[Note: there are literally an infinite number of ways to answer this question (correctly!); take pity on your poor instructor when choosing your parametrization....]

23. [BC#3.42.1(part)] Find the integrals $\int_C \frac{z+2}{z}\ dz$, where

(a): C is the semicircle $z = 2e^{i\theta}$, $0 \le \theta \le \pi$
(c): C is the circle $z = 2e^{i\theta}$, $0 \le \theta \le 2\pi$

24. [BC#4.42.8] Find the integral $\int_C z^n(\overline{z})^m\ dz$, where C is the unit circle $|z| = 1$ traversed in a counterclockwise direction.

[Note: you will fnd it helpful to know that $\int_0^{2\pi} e^{ik\theta}\ d\theta$ is 0 if $k \neq 0$, and 2π if $k = 0$. You need not prove this.]

Extra credit: why does it not matter where we choose to start our parametrization of the circle (i.e., at what point along the circle)?