Math 423/823 Homework 1

Due Thursday, Jan. 25

1. [BC#1.2.6(b)] For complex numbers $z_1 = a_1 + b_1 i$, etc., verify the distributive law:

$$z_1(z_2 + z_3) = z_1 z_2 + z_1 z_3$$

[N.B.: It's probably "better" to write things as $z_1 = (a_1, b_1)$, etc. and expand out both sides, to avoid inadvertently 'assuming' something?]

2. [BC#1.3.1] Reduce each of the quantities to a real number (the textbook gives the answers, so *show how you did it!*):

(a) $\dfrac{1+2i}{3-4i} + \dfrac{2-i}{5i}$ (c) $(1-i)^4$

3. [BC#1.5.8] Show that for any complex numbers $z_1, z_2 \in \mathbb{C}$, we have

$$|z_1 z_2| = |z_1| \cdot |z_2|.$$

- [This can be done directly ($z_1 = a_1 + b_1 i$, etc.), or you can appeal to polar/exponential form(s)...]

4. [BC#1.6.3] Show that for any complex numbers $z_1, z_2 \in \mathbb{C}$, we have

$$\overline{(z_1 z_2)} = \overline{z_1} \cdot \overline{z_2}.$$

- [Again, this can be done directly ($z_1 = a_1 + b_1 i$, etc.), or you can appeal to polar/exponential form(s)!]

5. Using the approach described in Section 1.11, find the (three!) complex numbers z with $z^3 = i$, writing them in both exponential ($re^{i\theta}$) and rectangular ($a + bi$) forms. [For the rectangular forms, evaluate (exactly) any trig function values that your answer contains!]