Math 423/823 Exercise Set 2

Due Thursday, Feb. 1

6. [BC#1.9.8] Show that for complex numbers z_1 and z_2 we have $|z_1| = |z_2|$ if and only if there are complex numbers c_1 and c_2 so that $z_1 = c_1c_2$ and $z_2 = c_1\overline{c_2}$.

   [N.B.: For $\Leftarrow$, a previous problem helps! For $\Rightarrow$, show that if $|z_1| = |z_2|$ then we can write $z_1/z_2 = e^{i\theta}$, and use this to construct an appropriate $c_2 = e^{i\psi}$, by looking at what z_1/z_2 <u>would</u> be if the conclusion were true! Or you can unravel the hints provided in the text...]

7. [BC#2.14.3] If $z = x + yi$ and $f(z) = (x^2 - y^2 - 2y) + (2x - 2xy)i$, use the formulas
$$x = \frac{z+\overline{z}}{2} \quad \text{and} \quad y = \frac{z-\overline{z}}{2i}$$
   to write $f(z)$ in terms of z (and $\overline{z}$) and simplify the result.

8. [BC#2.14.8] Sketch the regions onto which the sector
$$A = \{z = re^{i\theta} : 0 \le r \le 1, 0 \le \theta \le \pi/4\}$$
   of the complex plane is mapped by the functions

   (a) $w = z^2$ (b) $w = z^3$ (c) $w = z^4$

9. Show that the reciprocal function, $f(z) = 1/z$, maps the (punctured) disk
$$D = \{z : |z-1| < 2 \text{ and } z \ne 0\}$$
   onto the region that lies <u>outside</u> of the circle $\{w : |w + 1/3| = 2/3\}$.

   [N.B. Essentially, this is asking you to show that $|z-1| < 2$ (and $z \ne 0$) $\Leftrightarrow$ $|1/z + 1/3| > 2/3$.

   The quickest way that your instructor found to do this was to start with $z = x + yi$ and $|1/z+1/3|^2 > (2/3)^2$, clear the denomenator and continue to simplify the expression until he was staring at $|z-1|^2 < 4$ ...]