Math 423/823 Exercise Set 3

Due Thursday, Feb. 8

10. Express the function $f(x+yi) = f(z) = \dfrac{z^2+1}{z+1}$ as $f(z) = u(x,y) + iv(x,y)$ with $u(x,y), v(x,y) \in \mathbb{R}$ (i.e., the real and imaginary parts of f).

11. Find $\lim\limits_{z \to 1+i} \dfrac{z^2+z-1-3i}{z^2-2z+2}$.

12. [BC#2.18.7,p.55 (sort of)] Show that if $\lim\limits_{z \to z_0} f(z) = L$, then $\lim\limits_{z \to z_0} \overline{f(z)} = \overline{L}$ and $\lim\limits_{z \to z_0} |f(z)| = |L|$.

    [Ignore the hint the book provides! Instead think in terms of Theorem 1 of Section 16 (about real and imaginary parts of f) and things you know from Calc 1/2/3... That's the point to establishing theorems in the first place!]

13. [BC#2.20.9,p.62] Let f be the function $f(z) = \begin{cases} (\overline{z})^2/z & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$.

    Show that this function is continuous everywhere (Problem #12 will help!). Show, however, that f is not differentiable at 0, even though the limit of the difference quotient exists (and both agree) when you let $\Delta z \to 0$ along the vertical and horizontal axes; show that if you approach 0 along the line $h = k$ (where $\Delta z = h + ik$) you find a different limit.